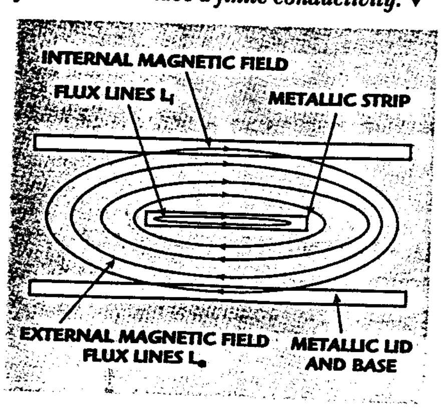
Transmission Line Conductor Loss and the Incremental Inductance Rule

Transmission line conductor loss calculations are often done using the incremental inductance rule. Although this useful technique has been around for many years and is used in commercial computer-aided design (CAD) programs, unfortunately it is often not understood by the current generation of design engineers, who are often faced with using transmission line structures, such as multiple dielectric layer transmission lines, that some CAD programs will not analyze. In addition, an understanding of the incremental inductance rule aids in the physical understanding of transmission lines and the loss mechanisms at work in them. This paper discusses the incremental inductance rule. A simple derivation is presented and design equations are shown.

Transmission line design usually entails specifying line impedance, effective dielectric constant (propagation constant) and line loss. A numerical technique can be used to calculate the desired data. However, usually a commercial CAD program is chosen. The benefit of these programs is the ease and speed at which transmission lines can be designed. The drawback is a tendency to lean too heavily upon them, resulting in a lack of understanding of the basis of the calculations. This scenario is especially true of line loss calculations, which can result in a lack of insight into the physics of transmission lines. The incremental inductance rule is used by many of the commercially available microwave CAD programs to calculate the conductor loss contribution to the overall transmission line loss (there are also dielectric losses). Understanding this practical and useful technique can add to the insight into transmission lines.

***Fig. 1** Magnetic fields penetrate into metal features that have a finite conductivity.*

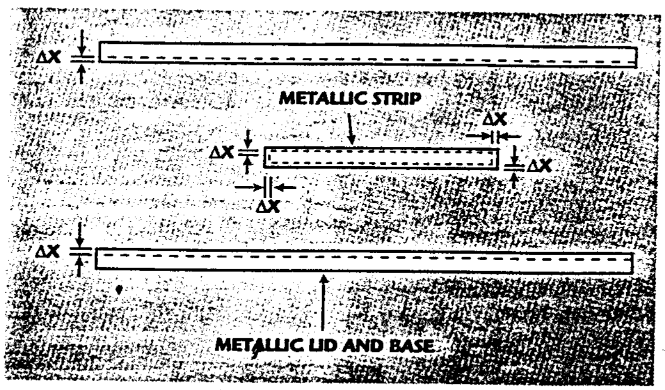


The incremental inductance rule was developed by H.A. Wheeler in 1942.[1,2] As a starting point to understanding this technique, consider the sources of inductance in a transmission line. **Figure 1** shows a microstrip line that uses slightly lossy conductors. There are magnetic fields existing within the conductors (as a result of the lossy metal) and the surrounding space. The internal inductance results from the penetration of the magnetic field into the conductor. This internal inductance can be used to find the conductor loss of the transmission line.

[Continued on page 158]

Rick Sturdivant
TRW
Redondo Beach, CA

Fig. 2 Incremental recession of the walls for stripline. ▼



DERIVATION

Direct computation of the internal inductance of a transmission line can be difficult. Wheeler's contribution showed how the internal inductance can be found from the external inductance and how it can be related to conductor loss. The internal inductance L_i can be found by the amount of increase in the external inductance L_e when the metal thickness is incrementally reduced. This process is of immense benefit and practical use since simple transmission line analysis methods can be used to find the external inductance.

Figure 2 shows stripline that has several metal faces, each of them being incrementally reduced by Δx. To derive Wheeler's equation, initially only a single metal face is considered. If the external inductance of the transmission line as a function of a metal face is given by $L_e(x)$, and the metal dimension is reduced by Δx, the resulting external inductance would be given by $L_e(x - \Delta x)$. This external inductance can be expanded in a Taylor series[2] expressed as

$$L_e\left(x - \frac{\mu}{\mu_0}\frac{\delta}{2}\right) \approx L_e - \frac{\mu}{\mu_0}\frac{\delta}{2}\frac{\partial L_e}{\partial x} \qquad (1)$$

where Δx has been replaced b $(\mu/\mu_0)(\delta/2)$, which is half the penetr tion into the metal. This equatio states that the value of L_e evaluated small amount less than x is equal t the value at x, stated as $L_e(x)$, minu the slope of L_e multiplied by th small increment. This relationshi can be loosely interpreted to physi cally mean that if a metal face in transmission line is reduced by an in cremental amount, the external in ductance of that transmission line i given by the external inductance be fore the metal face was reduced mi nus the internal inductance due tc penetrating magnetic fields withir the metal. Rearranging Equation 1 leads to a formula that is close to the desired one. The internal inductance is given by the difference between the external inductance before and after the metal face is incrementally reduced.

$$L_i = L_e(x) - L_e\left(x - \frac{\mu}{\mu_0}\frac{\delta}{2}\right) \approx \frac{\mu}{\mu_0}\frac{\delta}{2}\frac{\partial L_e}{\partial x} \qquad (2)$$

Now that the internal inductance has been found as a function of the external inductance, the next step is to relate it to the conductor loss. To accomplish this, the current flow is considered within the metal that generates the internal magnetic field. The current flow causes a complex surface impedance that is given as

$$Z = R + j\omega L_i \qquad (3)$$

This complex surface impedance has equal resistive and reactive components, which can be shown by integration of the current density.[1] Therefore, combining the internal inductance formula (Equation 2) with the surface impedance equation (Equation 3) gives

$$R = \omega L_i = \frac{R_s}{\mu_0}\frac{\partial L_e}{\partial x} \qquad (4)$$

where

R_s = surface resistance
$= \omega\mu\frac{\delta}{2}$

Equation 4 gives the effective or skin resistance of the transmission line

[Continued on page 160]

taking into account the high frequency skin depth effects. Finally, the transmission line attenuation due to conductor loss is given by

$$A_c = \frac{\text{power loss due to conductors}}{\text{two} \bullet \text{power transmitted}} = \frac{|I|^2 R}{2|I|^2 Z_0} = \frac{R_s}{2\mu_0 Z_0}\frac{\partial L_e}{\partial x} \text{ nepers/m} \quad (5)$$

Equation 5 gives the conductor loss per unit length for any transmission line for which the rate of change of external inductance can be found with the recession of the metal surface. This approach should be valid for all exposed metal features that are at least twice as large as the skin depth. This specification applies to metal thickness and radius of curvature of the metal.

APPLICATION

In applying this method, Equatio 5 is expanded to allow for the dime sions of each metal face to vary incr mentally and for unique metal co ductivities at each metal face. The a tenuation is the sum of the loss fro each conductor face. Therefor Equation 5 becomes

$$A_c = 8.686 \frac{1}{2\mu_0 Z_0} \sum_{i=1}^{m} R_{si} \frac{\partial L_e}{\partial x_i} = 8.686 \frac{\sqrt{\varepsilon_{r,\,eff}}}{2\eta_0 Z'_0} \sum_{i=1}^{m} R_{si} \frac{\partial Z'_0}{\partial x_i} \text{ dB/m} \quad (6)$$

The 8.686 factor converts fro nepers to dB. Also, $\partial L_e/\partial x_i$ signifie partial differentiation of external in ductance with respect to metal su face i, and $\partial Z'_0/\partial x_i$ signifies partia differentiation of the line impedanc (with the substrates replaced by air with respect to metal surface i. R_{si} i the surface resistance of the i^{th} meta surface and x_i is normal to the i^t metal face. ***Appendix A*** defines th variables used in the derivation.

In practice, the derivative of th line impedance for the utilized trans mission line is evaluated instead o the inductance. This procedure as sumes that a closed form equation ex ists that takes into account finite lin thickness, which has been done fo many popular transmission lines. ***Appendix B*** lists the conductor los equations for coax and twin lea transmission lines. Loss formulas fo stripline, microstrip and CPW have been described.[5] Equation 6 is readily applied to transmission lines where reliable closed form equations for line impedance (or equivalent) exist that take into account all metal features, including finite strip thickness. However, such equations do not always exist.

This method can also be applied to more exotic transmission lines. For instance, the quasi-static variational technique can be used with the incremental inductance formula. By modifying the spatial Green's function to account for finite line thickness and numerically incrementing the conductor faces, the conductor loss can be found.[4,5] A computer program with this capability for a transmission

[Continued on page 162]

line with multiple dielectric layers and a single conducting strip was developed. It will calculate line impedance, effective dielectric constant and conductor loss.

CONCLUSION

The incremental inductance rule has been described. The derivation was presented along with a physical explanation of the equations where appropriate. In addition, equations were shown that give the conductor loss for some popular transmission lines. The FORTRAN source code capable of calculating line impedance, effective dielectric constant, conductor loss and dielectric loss for a four-dielectric-layer transmission line is available from the author. ■

References

1. H.A. Wheeler, "Formulas for the Sk fect," Proceedings of the IRE, Septe 1942, pp. 412–424.
2. L.N. Dworsky, *Modern Transmission Theory and Applications*, Krieger, 19
3. B. Bhat and K.S. Koul, *Stripline Transmission Lines for Microwave grated Circuits*, Halsted Press, 1990.
4. B. Bhat and S.K. Koul, "Unified App to Solve a Class of Strip and Micro like Transmission Lines," IEEE Trans crowave Theory Techniques, Vol. MT May 1982, pp. 679–686.
5. K.C. Gupta, R. Garg and R. Chadha, C *puter-aided Design of Microwave Circ* Artech House, Norwood, MA, 1981.

APPENDIX A: VARIABLE DEFINITIONS

$\delta = \frac{1}{\sqrt{\pi\mu\sigma f}}$

= skin depth

σ = metal conductivity

λ_0 = free space wavelength

$R_s = \omega\mu\frac{\delta}{2}$

= surface resistivity

R = effective resistance (due to skin effects)

Z'_0 = line impedance (with dielectric removed)

Z_0 = line impedance

$\varepsilon_{r,eff}$ = effective dielectric constant

L_e = external line inductance

L_i = internal line inductance

$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

= impedance at free space

APPENDIX B: EQUATION FOR COAX

$$A_c = 13.64\frac{\delta\sqrt{\varepsilon_r}}{\lambda_0 \ln\left(\frac{b}{a}\right)}\left[\frac{1}{b}+\frac{1}{a}\right] \text{dB/length}$$

where:

a = radius of center conductor

b = radius of outer conductor

EQUATION FOR TWIN LEAD

$$A_c = 27.29\frac{\delta\sqrt{\varepsilon_r}}{a\lambda_0 \ln\left(\frac{s}{a}\right)} \text{dB/length}$$

where

a = radius of the conductors

s = separation between conductors

Transmission Line Conductor Loss and The Incremental Inductance Rule: A Tutorial

Rick Sturdivant, TRW, Automotive Electronics Group
One Space Park, Redondo Beach, California
e-mail: r.strudivant@ieee.org

Abstract

Transmission line conductor loss calculations are often done using the incremental inductance rule. Although this useful technique has been around for many years and is used in commercial CAD programs, it is often not understood by the current generation of design engineers. This is unfortunate since design engineers are often faced with using transmission line structures which some CAD programs will not analyze (i.e. multiple dielectric layer transmission lines). In addition, an understanding of the incremental inductance rule aides in ones physical understanding of transmission lines and the loss mechanisms at work in them. Therefore, a tutorial is presented on the incremental inductance rule. A simple derivation is presented, design equations are shown, and FORTRAN source code which implements this technique for a four layer strip transmission line is given.

Introduction

Transmission line design usually entails specifying line impedance, effective dielectric constant (propagation constant), and line loss. The design engineer could use a numerical technique to calculate the desired data. However, the engineer will usually choose a commercial CAD program. The benefit of these programs is the ease and speed at which one can design transmission lines. The drawback, of course, is a tendency to lean too heavily upon them and not understand the basis of the calculations. This is especially true of line loss calculations. This can result in a lack of insight into the physics of transmission lines. The incremental inductance rule is used by many of the commercially available microwave CAD programs to calculate the conductor loss contribution to the overall transmission line loss (there are also dielectric losses). This is a practically useful technique and an understanding of it can add to ones insight into transmission lines. The purpose of this article, then, is to be a tutorial on the incremental inductance rule for the new generation of designers and to those old timers who just never had time to learn it.

The incremental inductance rule was developed by H. A. Wheeler in 1942 [1,2]. As a starting point to understanding this technique, consider the sources of inductance in a transmission line. Figure 1 shows a microstrip line which uses slightly lossy conductors. Notice that there are magnetic fields existing within the conductors (as a result of the lossy metal) and the surrounding space.. The internal inductance results from the penetration of the magnetic field into the conductor. This internal inductance can be used to find the conductor loss of the transmission line.

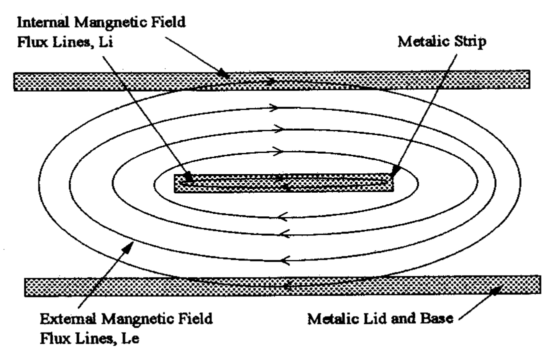


Figure 1. Magnetic fields penetrate into metal features which have a finite conductivity.

Derivation

Direct computation of the internal inductance of a transmission line can be difficult. Wheeler's contribution is that he showed how the internal inductance can be found from the external inductance and how it can be related to conductor loss. That is, he showed that the internal inductance, Li, can be found by the amount of increase in the external inductance, Le, when the metal thickness is incrementally reduced. This is of immense benefit and practical use since simple transmission line analysis methods can be used to find the external inductance.

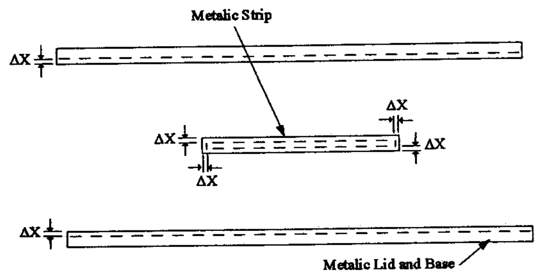


Figure 2. Incremental resession of the walls for stripline.

Figure 2 shows stripline which has several metal faces, each of them being incrementally reduced by Δx. To derive Wheeler's equation one initially considers only a single metal face. If the external inductance of the transmission line as a function of a metal face is given by Le(x), and the metal dimension is reduced by Δx, the resultant external inductance would be given by Le(x-Δx). This external inductance can be expanded in a Taylor series[2] shown below,

$$L_e\left(x-\frac{\mu}{\mu_o}\frac{\delta}{2}\right) \approx L_e(x) - \frac{\mu}{\mu_o}\frac{\delta}{2}\frac{\partial L_e}{\partial x} \quad (1)$$

where Δx has been replaced by $(\mu/\mu_o)(\delta/2)$ which, as pointed out by Wheeler, is half the penetration into the metal. This equation is telling us that the value of Le evaluated a small amount less than x is equal to the value at x, Le(x), minus the slope of Le multiplied by the small increment. This can be loosely interpreted to physically mean that if a metal face in a transmission line is reduced by an incremental amount, the external inductance of that transmission line is given by the external inductance before the metal face was reduced minus the internal inductance due to penetrating magnetic fields within the metal. By re-arranging equation (1), it will lead to a formula which is very close to the desired one. That is, the internal inductance is given by the difference between the external inductance before and after the metal face is incrementally reduced.

$$L_i = L_e(x) - L_e\left(x-\frac{\mu}{\mu_o}\frac{\delta}{2}\right) \approx \frac{\mu}{\mu_o}\frac{\delta}{2}\frac{\partial L_e}{\partial x} \quad (2)$$

Now that the internal inductance has been found as a function of the external inductance, the next step is to relate this to the conductor loss. This can be done by first considering the current flow within the metal which generates the internal magnetic field. The current flow causes a complex surface impedance which is given below.

$$Z = R + j\omega L_i \quad (3)$$

This complex surface impedance has equal resistive and reactive components. This can be shown by integration of the current density [1]. Therefore, combining the internal inductance formula (2) with the the surface impedance equation (3), one is able to show

$$R = \omega L_i = \frac{R_s}{\mu_o}\frac{\partial L_e}{\partial x} \quad (4)$$

where Rs is the surface resistance given by $R_s = \omega\mu\delta/2$. Equation (4) gives the effective or 'skin' resistance of the transmission line taking into account the high frequency skin depth effects. Finally, the transmission line attenuation due to conductor loss is given by

.

$$A_c = \frac{\textit{power}\text{ loss due to conductors}}{\text{2 power transmitted}}$$

$$= \frac{|I|^2 R}{2|I|^2 Z_0} = \frac{R_s}{2\mu_o Z_o}\frac{\partial L_e}{\partial x} \text{ nepers/m} \quad (5)$$

Equation (5) gives the conductor loss per unit length for any transmission line for which the rate of change of external inductance can be found with recession of the metal surface. This approach should be valid for all exposed metal features which are at least twice as large as the skin depth. This applies to metal thickness and radius of curvature of the metal.

Application

In applying this method, one expands equation (5) to allow for the dimensions of each metal face to incrementally vary and for unique metal conductivities at each metal face. The attenuation is the sum of the loss from each conductor face. Therefore, equation (5) becomes

$$A_c = 8.686 \frac{1}{2\mu_o Z_o} \sum_{i=1}^{m} R_{si} \frac{\partial L_e}{\partial x_i}$$

$$= 8.686 \frac{\sqrt{\varepsilon_{r,eff}}}{2\eta_o Z'_o} \sum_{i=1}^{m} R_{si} \frac{\partial Z'_o}{\partial x_i} \quad \text{dB/m} \qquad (6)$$

The 8.686 factor converts from nepers to dB. Also, $\partial Le/\partial x_i$ signifies partial diffentation of the external inductance with respect to metal surface i, and $\partial Z'_o/\partial x_i$ signifies partial differentation of the line impedance (with the substrates replaced by air) with respect to metal surface i. R_{si} is the surface resistance of the i^{th} metal surface, and x_i is normal to the i^{th} metal face. Tabel 1 gives definitions to the variables used in the derivation.

Variable Definitions	
$\delta = 1/\sqrt{\pi\mu\sigma f}$ = skin depth	Z_o = line impedance
σ = metal conductivity	$\varepsilon_{r,eff}$ = effective dielectric constant
λ_o = free space wavelength	L_e = external line inductance
$R_s = \omega\mu\delta/2$ = surface resistivity	L_i = internal line inductance
R = effective resistance (due to skin effects)	$\eta o = \sqrt{\mu_o/\varepsilon_o}$ = impedance of free space
Z'_o = line impedance (with dielectric removed)	

Table 1. Variable definitions.

In practice, one evaluates the derivative of the line impedance for the transmission line being evaluated, instead of the inductance. This, of course, assumes that a closed form equation exists which takes into account finite line thickness. This has been done for many popular transmission lines. Table 2 shows the conductor loss equations for coax and twin lead transmission lines. Reference [5] is an excellent source of loss formulas for stripline, microstrip, and CPW. Therefore, equation (6) is readily applied to transmission lines where reliable closed form equations for line impedance (or equivalent) exist which take into account <u>all</u> metal features, including finite strip thickness. However, such equations do not always exist.

Equations For Coax:

$$Ac = 13.64 \frac{\delta\sqrt{\varepsilon_r}}{\lambda_o \ln(b/a)} [1/b + 1/a] \quad \text{dB/length}$$

where:
a = radius of center conductor
b = radius of outer conductor

Equations For Twin Lead:

$$Ac = 27.29 \frac{\delta\sqrt{\varepsilon_r}}{a\,\lambda_o \ln(s/a)} \quad \text{dB/length}$$

where:
a = radius of the conductors
s = seperation between conductors

Table 2. Conductor loss equations for coax and twin lead transmission lines.

This method can also be applied to more exotic transmission lines. The quasi-static variational technique, for instance, can be used with the incremental inductance formula. By modifying the spatial Green's function to account for finite line thickness and numerically incrementing the conductor faces, one can find the conductor loss [4,5]. A computer program to do this for a transmission line with multiple dielectric layers and a single conducting strip was developed. It will calculate line impedance, effective dielectric constant, and conductor loss. Appendix A gives the FORTRAN source code.

Conclusions

A tutorial has been presented on the incremental inductance rule. The derivation was presented along with physical explanation of the equations where appropriate. In addition, equations were shown which give the conductor loss for some popular transmission lines. Finally, a FORTRAN program was presented which will calculate the line impedance, effective dielectric constant, conductor loss, and dielectric loss for a four dielectric layer transmission line.

References

[1] H.A. Wheeler, "Formulas for the skin effect," Proceedings of the IRE, Sept. 1942, pp 412-424

[2] L.N. Dworsky, Modern Transmission Line Theory and Applications, Krieger, 1988

[3] B. Bhat and K.S. Koul, Stripline-Like Transmission Lines for Microwave Integrated Circuits, Halsted Press, 1990.

[4] B. Bhat and S.K. Koul, "Unified approach to solve a class of strip and microstrip-like transmission lines," IEEE Trans. Microwave Theory Tech., Vol. MTT-30, pp. 679-686, May 1982.

[5] K.C. Gupta, R. Garg, and R. Chadha, Computer Aided Design of Microwave Circuits, Deham, Mass: Artech House, 1981.